

94 Solving Equations with a GC

AS 3.3
Trigonometric Graphs

Since trig equations have multiple solutions, the *Equation Solver* is not the best tool to solve them. Instead we will use the GRAPH mode with G-Solv options. The strategy is based on the fact that an equation of the form $f(x) = k$ can be solved by graphing $y = f(x)$, then calculate x for $y = k$.

Example : Solve $40 \cos(x + 30^\circ) = 25$, $-180^\circ \leq x \leq 80^\circ$

Method : Enter **GRAPH** and write the equation **Y=** $40 \cos(x + 30)$. In **SETUP** set the Angle to degrees; in **V-Window** make x go from -180 to 180 scale 30 , and y from -50 to 50 , scale 10 . (Note that some knowledge of graphs is needed to choose these!)

Now **DRAW** the graph, press **G-Solv** and scroll to **X-CAL**. Enter **Y=** 25 .

The first solution given will be $x = -81.3^\circ$ (1 dp). Press **▶** to get the next solution $x = 21.3^\circ$ (1 dp).

Answer : $x = -81.3^\circ$ (1 dp) or $x = 21.3^\circ$ (1 dp)

A Finding X-Values

1 Solve for x (angle in radians).

a) $5 \sin(x - \frac{\pi}{4}) = 3.15$, $-\pi \leq x \leq 3\pi$

b) $2 \tan \pi x = 8$, $0 \leq x \leq 4$

c) $\frac{1}{2} \cos(\frac{\pi}{2}x) = -0.2$, $0 \leq x \leq 4$

2a) Solve $20 - 5 \cos 2x = 22$, $0 \leq x \leq 2\pi$.

b) Explain why the equation $20 - 5 \cos 2x = 12$ has no solutions.

3 Give the set of x -values for which . . .

a) $-3 \sin 2x > 2$ and $0 \leq x \leq 2\pi$.

b) $8 - \cos(x + \frac{\pi}{3}) < 7.5$ and $-\pi \leq x \leq \pi$.

B Finding Features of Graphs

You can also use your GC to find x -intercepts (ROOTS) or turning points (MAX, MIN).

1 Consider the graph of $y = 3 \sin 2(x - 40^\circ)$ with domain $0 \leq x \leq 360^\circ$.

a) Give the coordinates of the maximum turning points.

b) Give the x -intercepts of the graph.

2 Given the function $y = 4 - 5 \sin 2x$ with domain $0^\circ \leq x \leq 360^\circ$.

a) Find the y -intercept.

b) For what values of x does the function reach its lowest value?

c) For what values of x is $4 - 5 \sin 2x < 0$?

3 Given the function $y = 2 \sec 3x$ on the interval $0 \leq x \leq 2\pi$.

a) Give the coordinates of the (local) maxima.

b) State the range of the function.

c) State the values of x where the function is undefined (i.e. the graph has an asymptote).

A Keeping Warm

The frequency of a periodic curve is the number of full cycles per unit on the x -axis. Usually frequency is asked when we have a function of time, in which case it is the number of cycles per time unit. If k is the period of the curve, then frequency is $f = \frac{1}{k}$.

- 1 A room has a heater with a thermostat which is set to 22°C . The actual temperature (T) in the room can be modelled by a cosine curve with equation $T(t) = 22 - 2 \cos(6\pi t)$, where t is the number of hours since 7 am.

a) What is the range of temperatures in the room?

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b) Calculate the period of the curve.

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c) Calculate the frequency. Explain the meaning of this number in context.

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d) Use your GC to find the first time after 7 am that the temperature reaches 23°C .

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e) Give all times between 7 am and 8 am that the temperature is exactly as set.

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C Puppet on a Spring

- 1 A puppet at the end of a spring is oscillating up and down about its rest position. The distance below the ceiling is modelled by $d = 45 + 20 \cos(\pi t)$, d in cm, t in seconds.

a) How far is the puppet below the ceiling when time starts?

b) What is the time needed to complete one oscillation?

c) A child can catch the puppet when it is at least 35 cm below the ceiling. How long per cycle can the child catch the puppet?

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d) Comment on the accuracy of the model.

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B Medication

- 1 A hospital patient is taking medication to reduce the amount of a harmful chemical in his blood stream. The amount of chemical still present can be modelled with the equation $A(t) = 1 + \sin \frac{\pi}{3}(t + 1)$. (t measured in hours after taking the first pill of the day.)

a) After how many hours is the amount down to zero for the first time?

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b) It takes a while for the medication to take effect. How long after taking the first pill is the amount of harmful chemical still increasing?

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c) When do you think the patient has another pill? Give a reason for your answer.

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d) Calculate the frequency of the curve.

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e) How many pills does the patient have in 24 hours?

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- A1 a) $x = -3.04'$ or 1.47 or 3.25 or 7.75
 b) $x = 0.42$ or 1.42 or 2.42 or 3.42
 c) $x = 1.26$ or 2.74
- A2 a) $x = 0.99$ or 2.15 or 4.13 or 5.29
 b) 12 is not in the range $15 \leq y \leq 25$
- A3 a) $1.94 < x < 2.78$ or $5.08 < x < 5.92$
 b) $-2.05 < x < 0$
- B1 a) $(85', 3)$ and $(265', 3)$
 b) $x = 40'$ or $130'$ or $220'$ or $310'$
- B2 a) 4 b) $x = 45'$ or $x = 225'$
 c) $26.6' < x < 63.4'$ or $208.6' < x < 243.4'$
- B3 a) $(1.05, -2)$ $(3.14, -2)$ $(5.24, -2)$
 b) $y \geq 2$ $y \leq -2$
 c) (in between turning points)
 $x = 0.52, x = 1.57, x = 2.62, \text{ etc}$

Page 95 - Using a Given Model

- A1 a) $20'$ to $24'$ b) $\frac{1}{3}$ hour (20 minutes)
 c) $f = 3$, there are 3 cycles per hour
 d) $x = 0.1 = \frac{1}{10}$ hour; time 7.07 am
 e) 7.05 am, 7.15 am, 7.25 am, 7.35 am, 7.45 am, 7.55 am
- B1 a) 3.5 hours (exact) b) for half an hour
 c) student's own answer d) $f = \frac{1}{6}$ cycle per hour
 e) 4 cycles per 24 hours 4 pills
- C1 a) 65 cm b) 2 seconds
 c) solve $d \geq 35$ from $x = 1\frac{1}{3}$ to $x = 2\frac{2}{3}$
 that is $1\frac{1}{3}$ seconds per cycle.
 d) Not very accurate since the motion will not continue at this constant rate, the amplitude will change.